Exercise 61

Prove, without graphing, that the graph of the function has at least two x-intercepts in the specified interval.

$$y = \sin x^3, \quad (1, 2)$$

Solution

The function $f(x) = \sin x^3$ is a composition of two functions, $g(x) = \sin x$ and $h(x) = x^3$, which are both continuous everywhere by Theorem 7. And by Theorem 9, $f(x)$ is continuous everywhere. Evaluate the function at several values of x in the interval of interest.

$$f(1) \approx 0.841$$

$$f(1.2) \approx 0.988$$

$$f(1.4) \approx 0.387$$

$$f(1.6) \approx -0.816$$

$$f(1.8) \approx -0.436$$

$$f(2) \approx 0.989$$

$f(x)$ is continuous on the closed interval $[1.4, 1.6]$, and $N = 0$ lies between $f(1.4)$ and $f(1.6)$. By the Intermediate Value Theorem, then, there exists an x-intercept within $1.4 < x < 1.6$. Also, $f(x)$ is continuous on the closed interval $[1.8, 2]$, and $N = 0$ lies between $f(1.8)$ and $f(2)$. By the Intermediate Value Theorem, then, there exists another x-intercept within $1.8 < x < 2$. Therefore, there are at least two x-intercepts in the interval $(1, 2)$—more can potentially be found by evaluating $f(x)$ at even more values of x.